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LINEAR STUDY OF ION ACOUSTIC WAVES IN MAGNETIZED PLASMA WITH CAIRNS DISTRIBUTED ELECTRONS

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ABSTRACT

The propagation of linear electrostatic waves is investigated in a magnetized plasma with Cairns distributed electrons and Maxwellian positrons. The dispersion relations are derived through linearization. It is found that the variation of several plasma parameters, like positron concentration p, warm ion effect σ, nonthermality parameter β and magnetic field strength Ω significantly alter the characteristic properties of ion acoustic waves.

1. INTRODUCTION

Over the last few decades, the study of linear and nonlinear wave phenomena in electron-positron-ion (EPI) plasma has gained a renewed interest and has been an active field for research. The pair production is the natural source of electron-positron plasmas occurring in high energy processes like in the early universe [1], neutron stars, active galactic nuclei [2] and in solar atmosphere [3]. The room-temperature plasma of 10^7 positrons with life of 10^3s produced via positron trapping technique has been reported by Greaves et al. [4]. Owing to this long life of positrons, most of the astrophysical and laboratory plasmas [5] have been an admixture of electrons, positrons, and ions. Therefore, the study of EPI plasmas is vital to grasp the behavior of astrophysical and laboratory plasmas.

The investigations of linear and nonlinear waves in various environments attract the attention of many researcher in the theoretical and experimental fields of plasma physics [6-25]. In the present work we study the linear phenomena by deriving the linear dispersion relation. Dispersion relation depends upon many properties of plasma, for example phase and space distribution functions of plasma particles, properties of plasma particles and the magnetic and electric fields. Some considerations are taken to obtain the dispersion relations for waves propagating in plasma. Warm homogeneous collision less plasma in a magnetic field is considered. Applying the kinetic theory and the linearization process to get dispersion relation which means that the initial point is a collision less (homogeneous) version of Boltzmann equations also called the Vlasov equations with the set of Maxwell's equations. We have three distinct forms [26,27] of the electrostatic waves or any modes of the pair ion plasma that can (IAW), ion plasma waves (IPW) and intermediate frequency waves (IFW). The IAW is a low frequency mode in which restoring force is produced by the pressure of electrons and the inertia is provided by the mass of ions. In the present study

we mainly focus to investigate the linear properties of IAW in magnetized EPI plasma composed of dynamic warm ions, Cairns distributed electrons and Maxwellian positrons.

The plan of the paper is as follows. In section 2 we present the basic set of equations on which the system depends, and the linear dispersion relation is derived. In section 3 we derive the linear dispersion (LD) equation. In section 4, the effect of variations of different parameters on the properties of IAW are discussed and summarized the results in the same section.

2. BASIC EQUATIONS

We consider a collisionless magnetized three component plasma consisting of warm ions, Cairns electrons [28] and thermal positrons. We assume that the plasma is immersed in a constant external magnetic field $B = B_0\hat{z}$. The set of equations governing the behavior ion acoustic waves may be described by the following set of normalized fluid equations;

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nv_x) + \frac{\partial}{\partial y}(nv_y) + \frac{\partial}{\partial z}(nv_z) = 0 \tag{1}$$

$$\frac{\partial v_x}{\partial t} + \left(v_x\frac{\partial}{\partial x} + v_y\frac{\partial}{\partial y} + v_z\frac{\partial}{\partial z}\right)v_x = -\frac{\partial \phi}{\partial x} + \Omega v_y - \frac{5\sigma}{3}\frac{\partial n}{\partial x} \tag{2}$$

$$\frac{\partial v_y}{\partial t} + \left(v_x\frac{\partial}{\partial x} + v_y\frac{\partial}{\partial y} + v_z\frac{\partial}{\partial z}\right)v_y = -\frac{\partial \phi}{\partial y} - \Omega v_x - \frac{5\sigma}{3}\frac{\partial n}{\partial y} \tag{3}$$

$$\frac{\partial v_z}{\partial t} + \left(v_x\frac{\partial}{\partial x} + v_y\frac{\partial}{\partial y} + v_z\frac{\partial}{\partial z}\right)v_z = -\frac{\partial \phi}{\partial z} - \frac{5\sigma}{3}\frac{\partial n}{\partial z} \tag{4}$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = (1+p)n_e - pn_p - n \tag{5}$$

$$n_e = (1 - \beta\phi + \beta\phi^2)e^{\phi} \quad \text{and} \quad n_p = e^{-\delta\phi} \tag{6}$$

Here n_0 is the equilibrium value which normalizes the ion number density n whereby the ion-acoustic speed $C_s = \sqrt{\frac{T_e}{m}}$ normalizes the ion fluid velocity v and the normalized electrostatic wave potential is $\phi = \frac{e\phi}{T_e}$. The space variable (x) and time variable (t) have been normalized by Debye length $\lambda_D = \sqrt{\frac{T_e}{4\pi n_0 e^2}}$ and inverse plasma frequency $\omega_{pi}^{-1} = \sqrt{\frac{m}{4\pi n_0 e^2}}$ respectively. $\Omega_i = \frac{eB_0}{mc}$ and $\Omega = \frac{\Omega_i}{\omega_{pi}}$, $\delta = \frac{T_e}{T_p}$ with T_e and T_p represents the temperature of electron and positron respectively, σ is the thermality of ions, and $p = \frac{n_{p0}}{n_{i0}}$ with $n_{i0}(n_{p0})$ represents the unperturbed number density of ion (positron). The components of ion fluid velocity along the x, y and z axes are, respectively, v_x, v_y and v_z.

3. LINEAR ANALYSIS

Linearizing Eqs. (1)-(5) by using standard procedure where the first-order quantities are proportional to $\exp(ik \cdot r - i\omega t)$

$$\begin{bmatrix} n \to 1 + \varepsilon n_1 \\ v_{x,y,z} \to \varepsilon v_{x1,y1,z1} \\ \phi \to \varepsilon \phi_1 \\ \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \to (ik_x, ik_y, ik_z) \\ \frac{\partial}{\partial t} \to -i\omega \end{bmatrix} \tag{7}$$

using (1) in Eqs. (1)-(5) thus we obtain the following equations in first order;

$$-i\omega n_1 + ik_x v_{x1} + ik_y v_{y1} + ik_z v_{z1} = 0 \tag{8}$$

$$-i\omega v_{x1} + ik_x \phi_1 - \Omega v_{y1} + \frac{5\sigma}{3} ik_x n_1 = 0 \tag{9}$$

$$-i\omega v_{y1} + ik_y \phi_1 + \Omega v_{x1} + \frac{5\sigma}{3} ik_y n_1 = 0 \tag{10}$$

$$-i\omega v_{z1} + ik_z \phi_1 + ik_y v_{y1} + \frac{5\sigma}{3} ik_z n_1 = 0 \tag{11}$$

$$-[k^2 + (1+p)(1-\beta) + p\delta]\phi_1 + n_1 = 0 \tag{12}$$

Solving Eqs. (8)-(12), finally we obtain the linear dispersion relation in the form;

$$\omega^4 - \omega^2 \left(\frac{k^2}{k^2+(1+p)(1-\beta)+p\delta} + \frac{5\sigma}{3}k^2 + \Omega^2\right) + \Omega^2 \left(\frac{k_z^2}{k^2+(1+p)(1-\beta)+p\delta} + \frac{5\sigma}{3}k_z^2\right) = 0 \tag{13}$$

where $k^2 = k_x^2 + k_y^2 + k_z^2 = k_\perp^2 + k_\parallel^2,\ k_\parallel^2 = k^2 \cos^2\theta = k_z^2,\ k_x^2 + k_y^2 = k_\perp^2$ are the wave vector in three dimensions, k_z and $k_x,\ k_y$ are the components of wave vector along and perpendicular to the magnetic field. Eq. (13) is quadratic in ω^2, hence we get the positive roots as

$$\omega_\pm^2 = \frac{1}{2}\left(\frac{k^2}{k^2+(1+p)(1-\beta)+p\delta} + \frac{5\sigma}{3}k^2 + \Omega^2\right) \pm \frac{1}{2}\sqrt{\left(\frac{k^2}{k^2+(1+p)(1-\beta)+p\delta} + \frac{5\sigma}{3}k^2 + \Omega^2\right)^2 - 4\Omega^2\left(\frac{k_z^2}{k^2+(1+p)(1-\beta)+p\delta} + \frac{5\sigma}{3}k_z^2\right)} \tag{14}$$

where $\omega_+$and $\omega_-$are defines respectively the fast and slow modes of electrostatic waves. ω_-defines an acoustic mode. for a very small value of k_z we obtain

$$\lim_{k_z \to 0} \left(\frac{\omega_-}{k_z}\right) = \sqrt{\frac{1}{(1+p)(1-\beta)+p\delta} + \frac{5\sigma}{3}} \quad (15)$$

On the same way when $k \to 0$ then the fast mode of electrostatic waves $\omega_+ \to \Omega^2$.

3.1 PARALLEL PROPAGATION

Consider the case of parallel propagation in which $k = k_z$ and $k_x = k_y = 0$, Eq. (13) will reduce to the form

$$\omega_{\parallel}^2 = \frac{k^2}{k^2+(1+p)(1-\beta)+p\delta} + \frac{5\sigma}{3}k^2 \quad (16)$$

It is clear from Eq.(16) that $\omega_{\parallel}^2$ does not depends on magnetic field.

3.2 PERPENDICULAR PROPAGATION

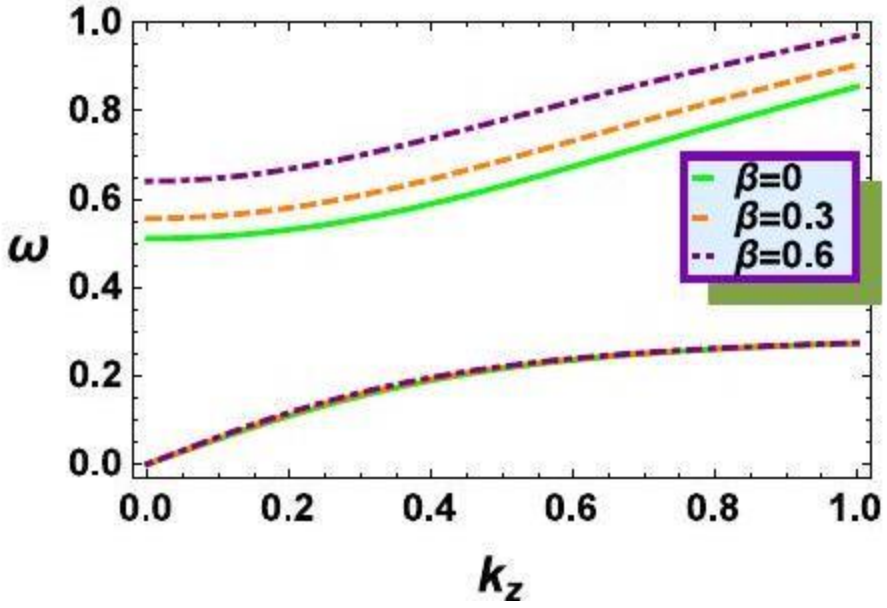
Now in this case we consider $k_z = 0$, Eq. (13) will take the form

$$\omega_{\perp}^2 = \frac{k^2}{k^2+(1+p)(1-\beta)+p\delta} + \frac{5\sigma}{3}k^2 + \Omega^2 \quad (17)$$

It is clear from Eq. (17) that $\omega_{\perp}^2$ is fuction of magnetic field strength Ω.

4. PARAMETRIC STUDY OF DISPERSION RELATION

To study the effect of plasma parameters such as no thermality parameter β, positron concentration p, ion thermality σ and the magnetic field strength Ω on the linear properties of ion acoustic waves. Dispersion relation are depending on these parameters through Eq. (14). The results are shown in Figs. $1(a) - 1(d)$. Fig. $1(a)$ shows that the frequency of both the slow and fast electrostatic modes increases when we increase the nonthermality parameter β. The effect of β is more visible in the fast mode as compared to slow mode. Fig. 1(b) displays that the frequency of both fast and slow modes decreases with increasing values of positron concentration p. The effect of ion thermality σ is dipected in Fig.1(c) which shows that the phase velocity of both fast and slow modes of ion acoustic waves increases when ion thermality σ is increased, the effect of ion thermality σ is more dominant in the fast mode as compared to slow mode. The effect of magnetic field is depicting in Fig.1 (d) which shows that the phase velocity of both fast and slow modes of ion acoustic waves increases when magnetic field Ω is increased.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0.6
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0.2
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ω
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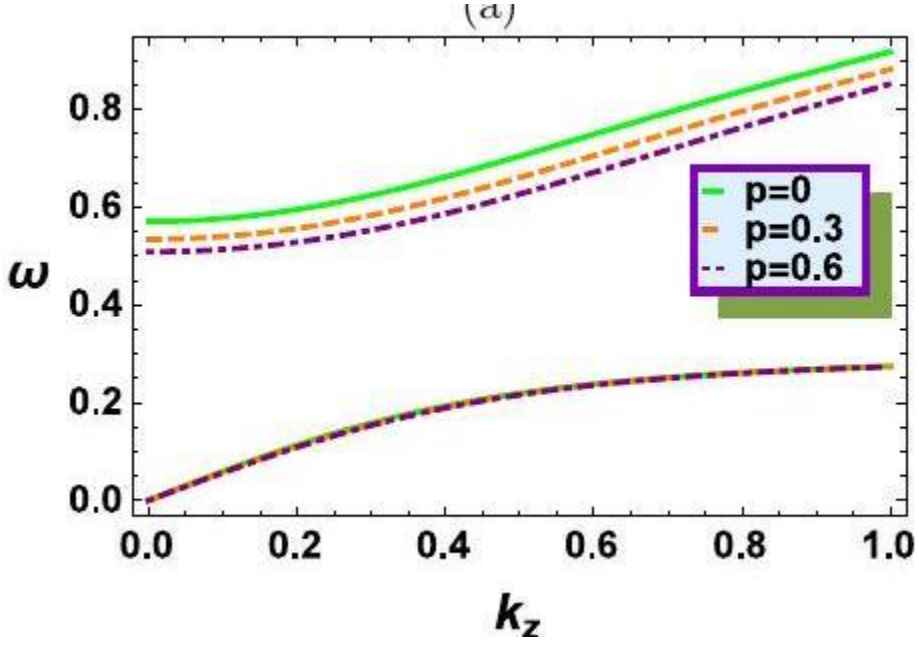
k_z
β=0
β=0.3
β=0.6

(a)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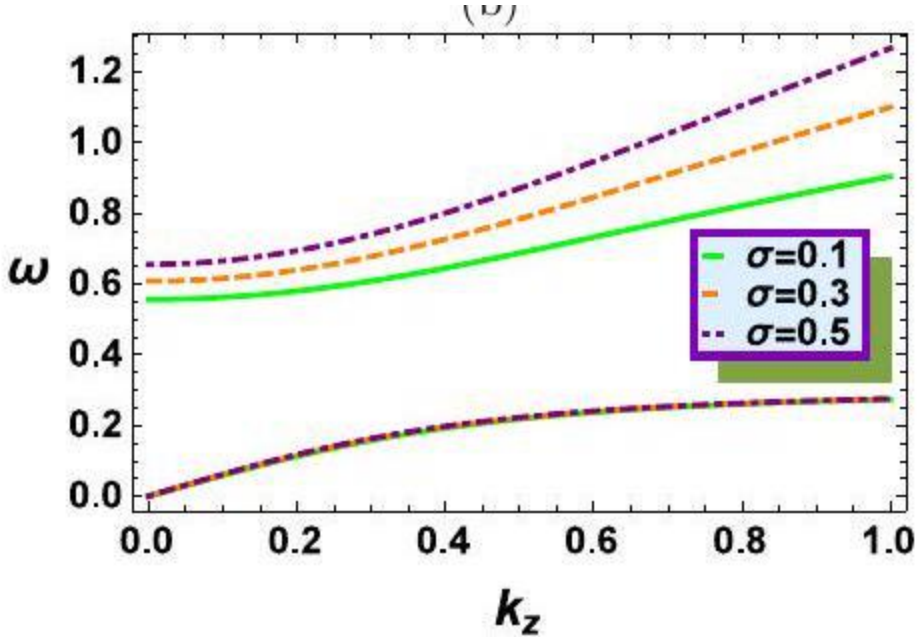
p=0.6

(b)



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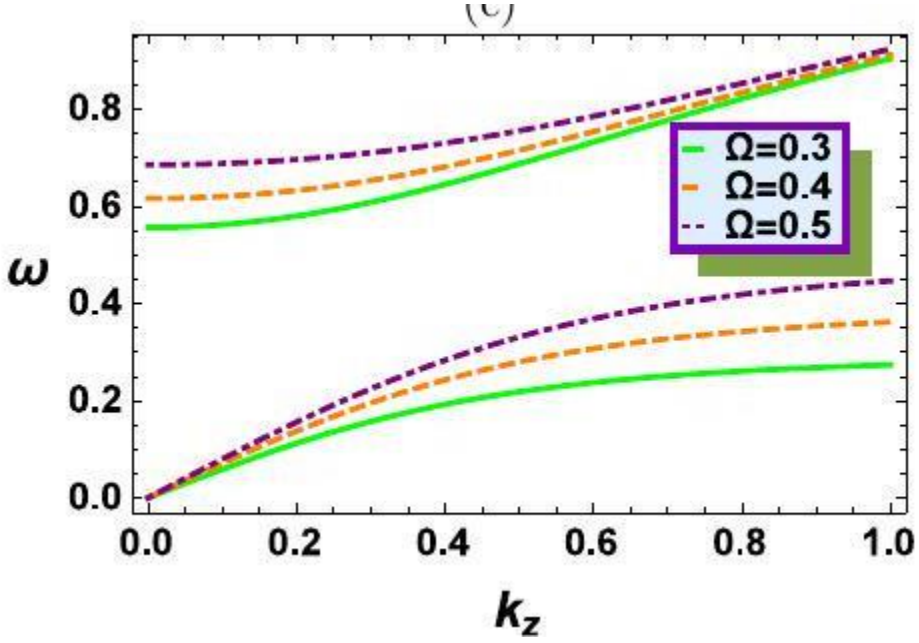
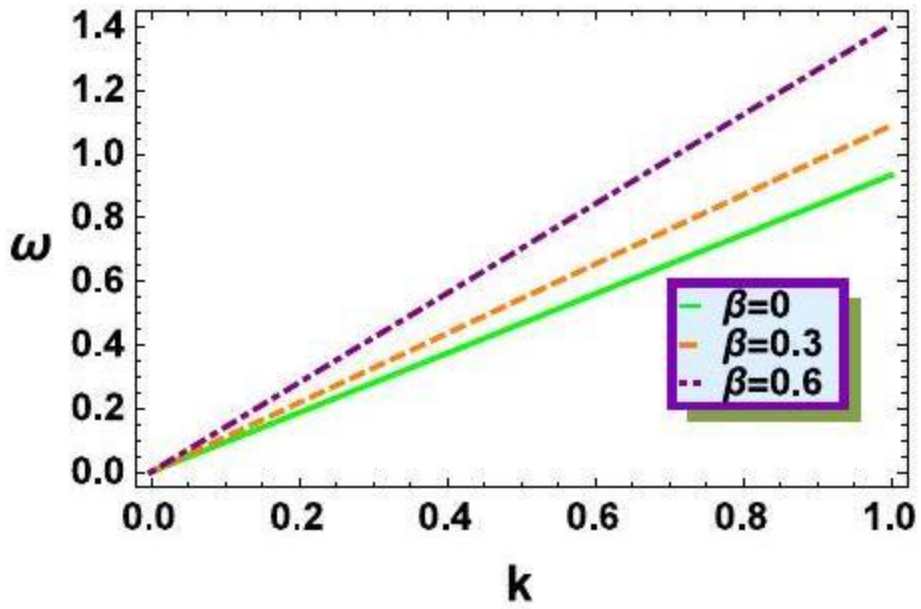


(d)

Figure 1: Graph of ω against k_z for $k_x = k_y = 0.3, \delta = 0.01$ (a) different values of β and $p = 0.1,\ \sigma = 0.1, \Omega = 0.3$, (b) different values of p and $\beta = 0.3, \sigma = 0.1, \Omega = 0.3$, (c) different values of σ and $p = 0.1, \beta = 0.3, \Omega = 0.3$ and (d) different values of Ω and $p = 0.1, \sigma = 0.1, \beta = 0.3$.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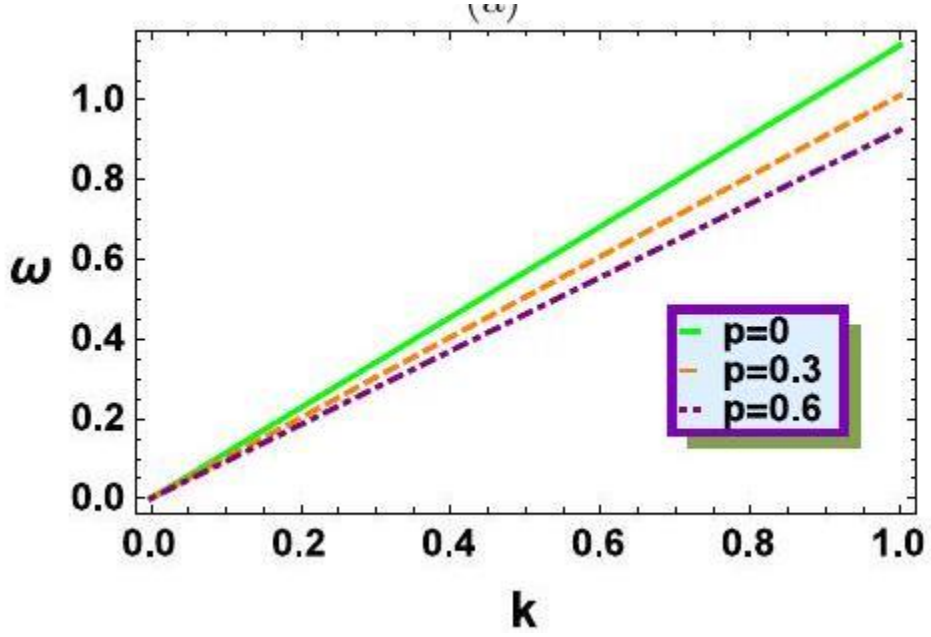
k
β=0
β=0.3
β=0.6

(a)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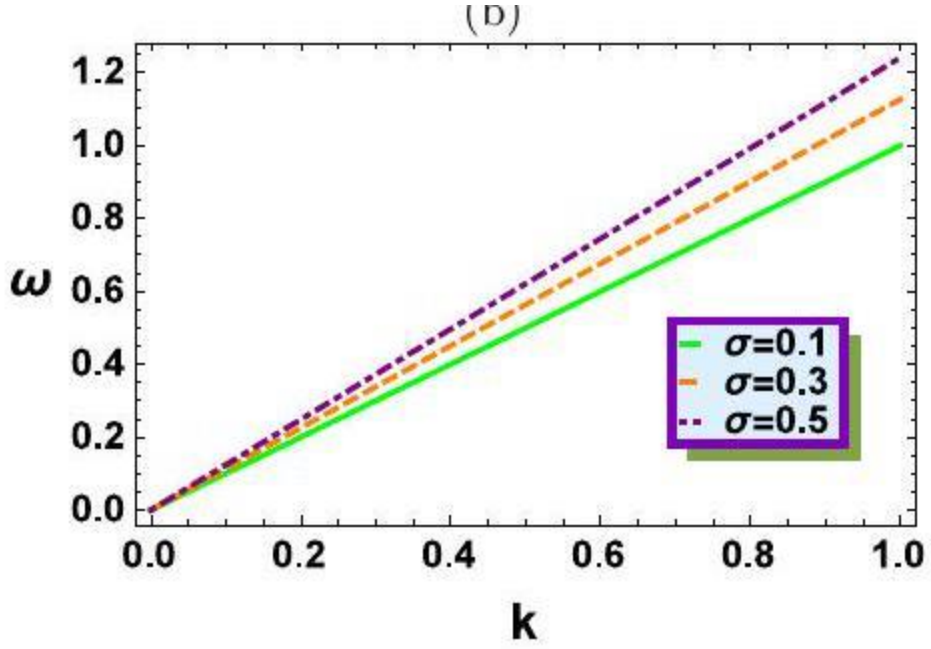
k
p=0
p=0.3
p=0.6

(b)



(C)

Figure 2: Graph of ω against k for $\theta = 25^0, \delta = 0.01$ (a) different values of β and $p = 0.1, \sigma = 0.1$, (b) different values of p and $\beta = 0.3, \sigma = 0.1$ and (c) different values of σ and $p = 0.1, \beta = 0.3$. phase speed of fast modes increases for small values of k_z with increasing Ω, and attains a constant asymptotic value for large k_z.It also shows that the frequency gap between the slow and fast ion acoustic modes decreases with magnetic field strength Ω at large k_z, while it increases for small values of k_z with magnetic field strength Ω.

In Fig. 2, we will now inquire the nature of the dispersion relation of the Eq. (15) considering different values of the positron concentration (via), non-thermality factor (β) and ion concentration effect (σ) for small values of " k". In Fig. 2(a), for increasing values of " " the frequencies tend to increase for increasing values of non-thermaility factor (β). In Fig. 2(b), it is shown that as we increase the positron concentration (p), the value of corresponding frequencies tends to increase. Similarly, in Fig. 2(c) as we increase warm ion effect (σ), the corresponding frequencies tend to increase.

Finally, we conclude that, the propagation of linear electrostatic waves is investigated in a magnetized plasma with Cairns distributed electrons and Maxwellian positrons. It is found that the variation of several plasma parameters, like the positron concentration p, non-thermaility factor β, warm ion effect σ and magnetic field Ω significantly alter the characteristic properties of IAW.

The frequency increases with higher values of k_z as the non-thermality factor (β) increases, that is for stronger deviations from Maxwellian case. Lowering of frequencies occur for fast as well as slow modes with increase in the positron concentration (p).The

acoustic mode increases the frequency as the warm ion effect (σ) is increased for both large and small values of k_z.

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